NOTE

On the Initial Flaw Sizes of uPVC Pipe Materials

It is well known that flaws in unplasticized poly(vinyl chloride) (uPVC) pipes are generated during processing. The flaws include inclusions, gas bubbles, and results from poor fusion between PVC grains and extrusion additives.[1,2] uPVC pipes are often used for water reticulation, subjected to an alternating pressure superimposed on a mean pressure. uPVC pipe samples have been found to fail in fatigue from a small semicircular flaw on the inside bore of the pipe.[3,4] In designing uPVC pipes against fatigue failure due to pressure variation, it is essential to determine the initial flaw size. Truss[5] attempted to estimate the lifespan of uPVC pipes and showed that predictions and experimental data are in good agreement.

In estimating the lifespan, Truss employed the following procedure: The fatigue crack growth data for uPVC were described by a modified Paris law equation:

$$\frac{da}{dN} = B(K_m\Delta K)^n \qquad (1)$$

where da/dN is the crack growth rate; K_m, the mean stress intensity factor; ΔK, the stress-intensity factor range; and B and n, constants. The number of cycles to failure (lifespan), N_f, can be calculated by integrating eq. (1), giving

$$N_f = \int_{a_0}^{a_c} \frac{da}{B(K_m\Delta K)^n} \qquad (2)$$

where a_0 is an inherent flaw size, and a_c, a critical flaw size. The K_m and ΔK are related to P_m and ΔP by

$$K_m\Delta K = FP_m\Delta Pa \qquad (3)$$

where $F = 1.562[(D + t)/2t]^2$, since

$$K^2 = \frac{3.77\sigma^2 a}{\phi^2 - 0.212(\sigma/\sigma_y)^2} \qquad (4)$$

where K is the stress intensity factor for a semicircular crack of depth, a, in a sheet subjected to a tensile stress, σ; ϕ is a semielliptical integral related to the shape of the crack ($\phi = 1.5708$ for a semicircular crack); and σ_y is the yield stress. The stress, σ, seen by the crack is due to both the hoop stress in the pipe, σ_h, and the pressure, P, opening up the crack:

$$\sigma = P\frac{D + t}{2t} \qquad (5)$$

where D is the outside diameter of the pipe, and t, the wall thickness.

In the above procedure, initial flaw sizes were assumed to be in a range between 0.5 and 1 mm. The purpose of this note was to examine the validity of the assumption made by Truss[5] on the flaw sizes.

For the purpose of the examination, the "Kitagawa plot"[6] may be employed as shown in Figure 1. The threshold stress range, $\Delta\sigma_{th}$, is schematically plotted as a function of crack length. If ΔK_{th} (threshold stress intensity factor range) is a material constant, the line with a slope of −0.5 can be drawn, since

$$\Delta K_{th} = \Delta\sigma_{th}\sqrt{\pi a} \qquad (6)$$

and

$$\log \Delta\sigma_{th} = -0.5 \log a + \log \frac{\Delta K_{th}}{\sqrt{\pi}} \qquad (7)$$

Equation (7) would predict an ever-increasing value of

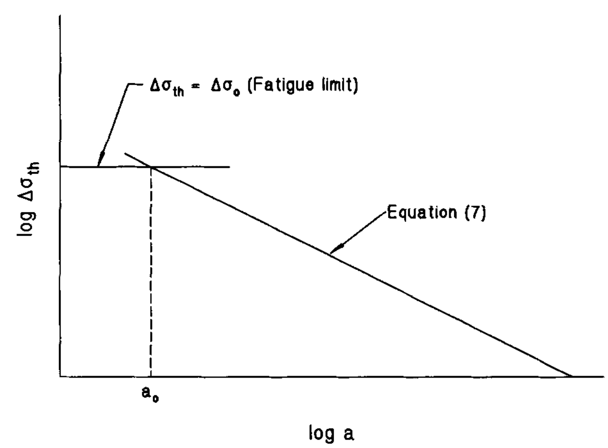

Figure 1 The "Kitagawa plot" of threshold stress range vs. crack length.

Journal of Applied Polymer Science, Vol. 50, 2223–2224 (1993)
 CCC 0021-8995/93/122223-02

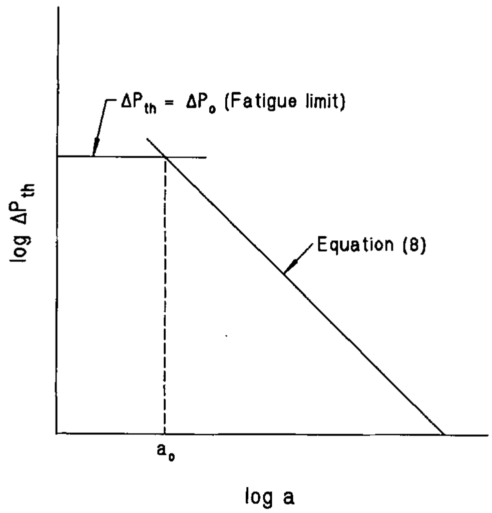

Figure 2 The "Kitagawa plot" with threshold pressure range as a function of flaw size after conversion.

$\Delta\sigma_{th}$ if crack length is decreased. However, we know that the maximum $\Delta\sigma_{th}$ is not infinity but is equal to the fatigue limit, $\Delta\sigma_0$. Therefore, we can draw a horizontal line at this value of $\Delta\sigma_0$ and then a_0 is determined. The a_0 may be regarded as the initial crack length. In practice, there would be a curved portion around the cutoff point so that the initial crack size would be slightly smaller than a_0. In a conservative sense, a_0 can be calculated for the initial flaw size.

To apply the principles of the "Kitagawa plot" in uPVC pipes, ΔP_{th} is obtained for values of threshold from eq. (3):

$$\Delta P_{th} = \frac{K_m \Delta K_{th}}{F P_m a} \quad (8)$$

where subscript th denotes threshold. Thus, eq. (8) allows us to construct a converted plot with a ΔP_0 (converted fatigue limit) as shown schematically in Figure 2. The values of a_0 in this plot can be obtained.

Calculations are conducted for 100 mm class 12 pipe.[5] The values for the fatigue threshold used for the calculations were obtained from Ref. 7: $\Delta K_{th} = 0.25 MPam^{1/2} - 0.2 R MPam^{1/2}$ for $0 \leq R \leq 0.5$ and $\Delta K_{th} = 0.15 MPam^{1/2}$ for $R \geq 0.5$, where R is the stress ratio. Also, values used for pressure ranges (which are in the range of fatigue limit) as well as calculated values of a_0 are listed in Table I. The calculated values of a_0 fall between 0.5 and 1 mm.

Table I Calculated Values of a_0 for Data Given in Ref. 5

P_m (MPa)	ΔP_{th} (MPa)	Cycles to Failure	Calculated a_0 (mm)
0.50	0.50	9.1×10^6	0.94
0.55	0.90	3.1×10^5	0.49
0.80	0.65	6.0×10^5	0.45
0.80	0.50	$> 6.4 \times 10^6$	0.63
1.00	0.30	$> 2.5 \times 10^7$	1.08

In conclusion, the assumption made by Truss on the initial flaw sizes of uPVC pipes appears to be valid.

REFERENCES

1. R. W. Truss, *Pure Appl. Chem.*, **57**, 993 (1985).
2. H. S. Kim, B. Cotterell, and Y. W. Mai, *Polym. Eng. Sci.*, **27**, 277 (1987).
3. S. H. Joseph and P. S. Leevers, *J. Mater. Sci.*, **20**, 237 (1985).
4. J. W. Stephens and B. W. Gill, in *Proceedings of Plastics and Pipes 5*, York, Sept. 1982 (PRI), Paper 33.
5. R. W. Truss, *Plast. Rubber Process. Appl.*, **10**, 1 (1988).
6. D. Tayler, *Fatigue Thresholds*, Butterworths, London, 1989.
7. H. S. Kim, Y. W. Mai, and B. Coterrell, *Polymer*, **29**, 277 (1988).

H. S. KIM
Department of Mechanical Engineering, The University of Newcastle, Newcastle, NSW 2308, Australia

Received December 19, 1992
Accepted April 17, 1993